system. Friction cancellation was dealt with in the pressure tracking case, and the Karnopp plus Stribeck friction model used for cancellation was verified. The experimental results showed that the proposed control law and adaptation scheme are effective for force/pressure tracking.

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Minimizing the Effect of Out of Bandwidth Modes in Truncated Structure Models

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The modal analysis approach to modeling of structures and acoustic systems results in infinite-dimensional models. For control design purposes, these models are simplified by removing higher frequency modes which lie out of the bandwidth of interest. Truncation can considerably perturb the in-bandwidth zeros of the truncated model. This paper suggests a method of minimizing the effect of the removed higher order modes on the low frequency dynamics of the truncated model by adding a zero frequency term to the low order model of the system. [S0022-0434(00)01501-X]

1 Introduction

Modal analysis approach has been extensively used throughout the literature to model dynamics of distributed parameter systems. Such systems include, but are not limited to, flexible beams and plates [1], slewing beams [2], piezoelectric laminate beams [3] and acoustic ducts [4]. These systems share the property that dynamics of each one of them is described by a particular partial differential equation. In the modal analysis approach the solution of these PDE's is assumed to consist of an infinite number of terms. Moreover, these terms are chosen to be orthogonal. Hence, the modal analysis modeling of a system can result in an infinite-dimensional model of that system.

In control design problems, one is often only interested in designing a controller for a particular frequency range. In these situations, one approach is to remove the modes which correspond to frequencies that lie out of the bandwidth of interest and only keep the low frequency modes. To improve the in-bandwidth response a number of out-of-bandwidth modes may also be kept. It is, of course, of interest to work with a low order model since modern controller design techniques result in controllers that are of the same dimension as that of the plant.

It is known that truncation has the potential to perturb the in-bandwidth zeros of the system. This problem is addressed in [5] and was recently revisited [6,7]. The mode acceleration method (see 350 of [5] and [6]) is concerned with capturing the effect of higher frequency modes on the low frequency dynamics of the system by adding a zero frequency term to the truncated model to account for the compliance of the ignored modes. In this paper, we allow for a zero frequency term to capture the effect of truncated modes. However, this constant term is found such that the $\mathcal{H}_2$ norm of the resulting error system is minimized.

To this end, we point out that there are alternative methods for modeling of distributed parameter systems. As an example, one can point to the recent works of Pota and Alberts in modeling of such systems using symbolic computations [8–10]. However, the models that are obtained via modal analysis have the interesting property that they describe spatial and temporal behavior of the system. Such models can then be used in designing spatial controllers [11–14].

2 Problem Statement

In general, modeling of a flexible structure via model analysis technique results in a model that can be represented by:

$$G(s)=\sum_{i=1}^{\infty}\frac{F_i}{s^2+\omega_i^2}. \tag{1}$$

This is an infinite-dimensional transfer function due to the existence of an infinite number of modes. We notice that Eq. (1) does not include any modal dampings. In reality, however, each mode is lightly damped. Therefore, a more precise version of Eq. (1) can be written as $G(s)=\Sigma_{i=1}^{\infty}F_i/s^2+2\zeta_i s+\omega_i^2$. It is a difficult task to determine modal structural dampings using physical principles. Therefore, ζ_i's are often determined by experiments. In this paper, we ignore the effect of modal dampings. However, it is straightforward to extend this work to include the effect of modal dampings.

In a typical control design scenario, the designer is often interested only in a particular bandwidth. Therefore, an approximate model of the system is needed that best represents the dynamics of the system in the prescribed frequency range. This is often done by truncating the model to

$$G_N(s)=\sum_{i=1}^{N}\frac{F_i}{s^2+\omega_i^2}. \tag{2}$$

A drawback of this approach is that the truncated higher order modes may contribute to the low frequency dynamics in the form of distorting zero locations [6]. This problem can be rectified, to some extent, by adding a zero frequency term to $G_N(s)$. That is,

$$\hat{G}(s)=G_N(s)+K \tag{3}$$

where $K=\Sigma_{i=N+1}^{\infty}F_i/\omega_i^2$. The logic behind this choice of K is that at lower frequencies one can ignore the effect of dynamical responses of higher order modes since they are much smaller than the forced responses at those frequencies.

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This paper is an attempt to find an optimal value for K. In other words, we will try to determine K such that the effect of higher order modes on the low frequency dynamics is minimized in some measure. Our objective here is to choose a value for K such that the following cost function is minimized,

$$\|(G(s)-\hat{G}(s))W(s)\|_2^2 \tag{4}$$

where $\|f(s)\|_2^2 = 1/2\pi\int_{-\infty}^{\infty}|f(j\omega)|^2 d\omega$. Here, $G(s)$ and $\hat{G}(s)$ are defined as in Eqs. (1) and (3) and $W(s)$ is an ideal low-pass weighting function with its cutoff frequency ω_c chosen to lie within the interval $\omega_c \in (\omega_N, \omega_{N+1})$. That is, $|W(j\omega)|=1$ for $-\omega_c \leq \omega \leq \omega_c$ and zero elsewhere. The reason for this choice of W will become clear soon. To this end, it should be clear that a K chosen to minimize Eq. (4) will minimize the effect of out of bandwidth dynamics of $G(s)$ on $\hat{G}(s)$ in an $\mathcal{H}_2$ optimal sense. Notice that the cost function (4) conveys no information on frequencies higher than ω_c.

It is easy to see that (4) is equivalent to

$$\left\| \left(\sum_{i=N+1}^{\infty} \frac{F_i}{s^2+\omega_i^2} - K \right) W(s) \right\|_2^2. \tag{5}$$

The fact that W is chosen to be an ideal low-pass filter with its cutoff frequency lower than the first out-of-bandwidth pole of G, guarantees that Eq. (5) will remain finite. Let $\tilde{G}(s) = \Sigma_{i=N+1}^{\infty} F_i / s^2 + \omega_i^2$. It is straightforward to show that Eq. (5) is equivalent to

$$\|\tilde{G}W\|_2^2 + K^2\|W\|_2^2 - K(\langle \tilde{G}W, W\rangle + \langle W, \tilde{G}W\rangle) \tag{6}$$

where $\langle f,g\rangle = 1/2\pi\int_{-\infty}^{\infty} f^*(j\omega)g(j\omega)d\omega$. It can be verified that the K that minimizes Eq. (6) is given by

$$K = \frac{\langle \tilde{G}W, W\rangle + \langle W, \tilde{G}W\rangle}{2\|W\|_2^2} \tag{7}$$

$$= \frac{\int_{-\infty}^{\infty} \mathrm{Re}(\tilde{G}(j\omega))|W(j\omega)|^2 d\omega}{\int_{-\infty}^{\infty}|W(j\omega)|^2 d\omega} \tag{8}$$

$$= \frac{\int_{-\infty}^{\infty}\left(\Sigma_{i=N+1}^{\infty} \frac{F_i}{\omega_i^2-\omega^2}\right)|W(j\omega)|^2 d\omega}{\int_{-\infty}^{\infty}|W(j\omega)|^2 d\omega} \tag{9}$$

where $\mathrm{Re}(f)$ represents the real part of the complex number f. Hence, to obtain the optimal K, one has to carry out the following integration.

$$K = \frac{1}{2\omega_c}\int_{-\omega_c}^{\omega_c} \sum_{i=N+1}^{\infty} \frac{F_i}{\omega_i^2-\omega^2} d\omega. \tag{10}$$

The optimal value of K is then found to be

$$K_{\mathrm{opt}} = \frac{1}{2\omega_c}\sum_{i=N+1}^{\infty} \frac{F_i}{\omega_i} \ln\left(\frac{\omega_i+\omega_c}{\omega_i-\omega_c}\right). \tag{11}$$

Next, we extend our model correction technique to multivariable transfer functions. This is an important issue since in many cases it may not be practical to achieve the required performance by a single actuator and sensor. If a multiple number of actuators and sensors are to be used, and the multivariable model is to be truncated, it is essential to capture the effect of higher order modes on the remaining in-bandwidth modes, as we did in the SISO case. In the multivariable case, the transfer function matrix of the system is given by:

$$G(s) = \sum_{i=1}^{\infty} \frac{1}{s^2+\omega_i^2}[F_i^{mn}]. \tag{12}$$

Here, $[F_i^{mn}]$ represents a matrix whose (m,n)th element is F_i^{mn}. Transfer function matrix $G(s)=[G^{mn}(s)]$ has an interesting property. All of its individual transfer functions share similar poles. However, the zeros can be different. Moreover, if the actuators and sensors are collocated, the diagonal transfer functions will possess minimum-phase zeros only. However, the off-diagonal transfer functions may have nonminimum-phase zeros since they correspond to noncollocated actuators and sensors.

It is our intention to approximate $G(s)$ by a finite number of modes, say N modes only. In this case, however, we choose to approximate the effect of higher order modes on the low-frequency dynamics of $G(s)$ by a constant matrix. That is, we approximate (12) by

$$\hat{G}(s) = \sum_{i=1}^{N} \frac{1}{s^2+\omega_i^2}[F_i^{mn}] + [k^{mn}]. \tag{13}$$

Let $K=[k^{mn}]$. We will determine K such that the following cost function is minimized:

$$J = \|W(s)(G(s)-\hat{G}(s))\|_2^2 \tag{14}$$

where for a multivariable $F, \|F(s)\|_2^2 = 1/2\pi\int_{-\infty}^{\infty}\mathrm{trace}\{F^*(j\omega) \times F(j\omega)\}d\omega$. Here, W is chosen to be a diagonal matrix, where the diagonal elements are ideal low-pass filters $W = \mathrm{diag}(w,w,\ldots,w)$ and w is an ideal low-pass filter as described above. The cost function (14) can be rewritten as $J=\|W(s) \times(\tilde{G}(s)-K)\|_2^2$ where $\tilde{G}(s)=\Sigma_{i=N+1}^{\infty} 1/s^2+\omega_i^2[F_i^{mn}]$. Therefore, $J=\|W\tilde{G}\|_2^2 + \|WK\|_2^2 - (\langle W\tilde{G}, WK\rangle + \langle WK, W\tilde{G}\rangle)$ where $\langle F,G\rangle = 1/2\pi\int_{-\infty}^{\infty}\mathrm{trace}\{F^*(j\omega)G(j\omega)\}d\omega$. The cost function can then be written as:

$$\begin{aligned} J = {} & \|W\tilde{G}\|_2^2 + \frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{trace}\{K' W(j\omega)^* W(j\omega) K\} d\omega \\ & - \frac{1}{2\pi}\int_{-\infty}^{\infty}(\mathrm{trace}\{\tilde{G}(j\omega)^* W(j\omega)^* W(j\omega) K\} \\ & + \mathrm{trace}\{K' W(j\omega)^* W(j\omega)\tilde{G}(j\omega)\})d\omega. \end{aligned}$$

Differentiating J with respect to K (see p. 592 of [15]), we obtain the optimum value of K.

$$\begin{aligned} K_{\mathrm{opt}} &= \left(\int_{-\infty}^{\infty} W(j\omega)^* W(j\omega) d\omega\right)^{-1} \\ &\quad \times \left(\int_{-\infty}^{\infty} W(j\omega)^* W(j\omega)\mathrm{Re}\{\tilde{G}(j\omega)\} d\omega\right) \\ &= \frac{1}{2\omega_c}\int_{-\omega_c}^{\omega_c}\mathrm{Re}\{\tilde{G}(j\omega)\}d\omega \\ &= \frac{1}{2\omega_c}\int_{-\omega_c}^{\omega_c}\sum_{i=N+1}^{\infty}\frac{1}{\omega_i^2-\omega^2}[F_i^{mn}]d\omega \\ &= \frac{1}{2\omega_c}\sum_{i=N+1}^{\infty}\frac{1}{\omega_i}\ln\left(\frac{\omega_i+\omega_c}{\omega_i-\omega_c}\right)[F_i^{mn}]. \end{aligned}$$

What this result implies is that one can use K_{opt} that was determined in Eq. (11) to approximate the effect of out-of-bandwidth modes on the individual truncated transfer functions of Eq. (12). The obtained multivariable transfer matrix will be optimal in the sense of Eq. (14). This is an interesting result which is mainly due to the fact that all individual transfer functions of Eq. (12) share similar poles.

To this end, we point out that this work does not address the issue of model parameter uncertainty and disturbances. Indeed, if there are uncertainties associated with modal parameters of the structures, the analysis presented in this paper has to be modified to accommodate such parameter deviations.

3 Example: A Simply-Supported Beam

In this section, we apply the approximation mechanism developed in Sec. 2 to a simple flexible structure. The structure consists of a flexible beam which is pinned at its both ends as shown in Fig. 1.

Here, $y(t,r)$ denotes the elastic deformation of the beam as measured from the rest position. The elastic deflection $y(t,r)$ is governed by the classical Bernoulli–Euler beam equation and its corresponding pinned boundary conditions. A transfer function for the beam can be found to be [1]

$$\frac{\hat{y}(s,r)}{U(s)}=\sum_{i=1}^{\infty}\frac{\phi_i(r_1)\phi_i(r)}{(s^2+\omega_i^2)} \tag{15}$$

where $\phi_i(r)=\sqrt{2/\rho AL}\,\sin(i\pi r/L)$ and the corresponding natural frequencies are $\omega_i=(i\pi/L)^2\sqrt{EI/\rho A}$.

Fig. 1 A simply supported flexible beam

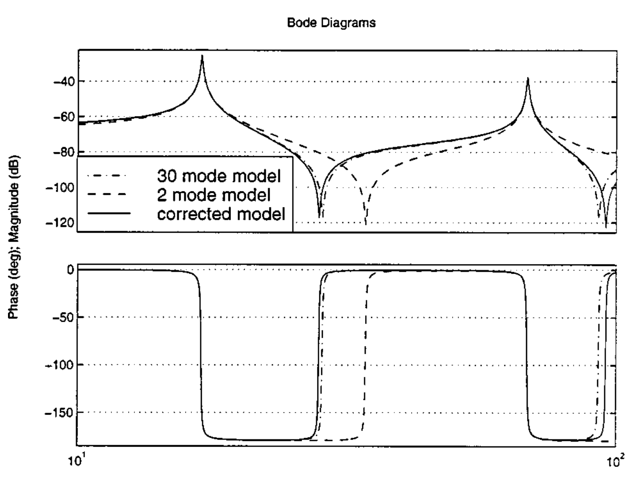

Fig. 2 Comparison of the frequency responses of the thirty mode model of the beam with its two mode model and a two mode model with a correcting zero-frequency term

Here, E, I, A, $u(t,r)$, and ρ represent, respectively, the Young's modulus, moment of inertia, cross-sectional area, external force per unit length, and the linear mass density of the beam. This system consists of an infinite number of modes and it describes the elastic deflection of the entire beam due to a point force applied at r_1.

The parameters of the beam are: L = beam length = 1.3 m, r_1 = 0.075 m, $r_2=r_1$, ρA = 0.6265 kg/m, EI = 5.329 Nm^2, where r_2 is the point at which the sensor is located. Since the actuator and the sensor are located at the same position, this is a collocated system.

In Fig. 2, we compare the frequency response of the two mode system and the system based on the first thirty modes in the frequency range of up to 100 rad/s, i.e., $\omega_c=100$ rad/s. Figure 2 also plots the corrected version of the two mode system based on the procedure developed in Section 2, i.e., by adding the optimal zero frequency term (11) to the two mode trucated model of the beam. The correction zero frequency term captures the effect of modes 3 to 30 on the two mode dynamics of the system. It can be observed that the corrected two mode system approximates the thirty mode system reasonably well in the frequency range of interest.

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