Modulated–demodulated control: Q control of an AFM microcantilever

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A B S T R A C T

We outline the application of modulated–demodulated control to the quality (Q) factor control of an atomic force microscope microcantilever. We review the modulated–demodulated control technique, emphasize its linear time invariant nature and develop state space representations of the controller for design and analysis. The modulated–demodulated controller can be configured as both positive position feedback (PPF) and resonant controllers, which are effective in the control of negative imaginary systems. Negative imaginary systems theory has important application in the control of collocated mechanical systems and we briefly summarize the key relevant results. A high-frequency, tunable modulated–demodulated controller, designed specifically for MHz operation, was developed for experimental validation. The modulated–demodulated controller enables the use of a low-bandwidth baseband controller in the configuration of a high-bandwidth controller, thus simplifying the implementation of high-bandwidth controllers. We outline the controller characterization and demonstrate closed-loop control of a Bruker DMASP microcantilever. We also present AFM images highlighting the improvements in scan speed and image quality achieved as a result of Q control. Modulated–demodulated control appears well suited to the control of high-frequency resonant dynamics. In addition to high-speed atomic force microscopy, we believe this control technique may find applications in high-frequency microelectromechanical systems (MEMS).

1. Introduction

Since their emergence, modulated–demodulated control systems have encountered application in radio receivers and the control of rotating systems, such as gyroscopes [1,2] and gravity gradiometry [3–5]. More recent examples have highlighted their application to the vibration control of a cantilever [6] as well as dynamic phase compensation, resulting in improved disturbance rejection at a specific frequency [7].

In an effort to redefine high-frequency control, we recently introduced the concept of modulated–demodulated control to applications in nanotechnology by demonstrating application of this method to the quality (Q) factor control of an atomic force microscope microcantilever [8]. A high-frequency modulated–demodulated positive position feedback (PPF) controller was developed and experimentally verified on a Bruker DMASP microcantilever, highlighting the potential of this technique in the control of high-frequency resonant dynamics.

Motivated by linear modulation techniques, modulated–demodulated control systems employ synchronous demodulation to shift measured high-frequency oscillations down to the baseband. Provided that the resonance frequency of the mode is significantly higher than its half-power bandwidth, the demodulation process extracts the slow time-varying amplitude envelope of the measured signal, thus significantly reducing the required bandwidth of the controller and enabling the use of low-bandwidth, reconfigurable controllers in the baseband to control high-frequency resonant dynamics. Modulation of the baseband controller outputs shifts the control signal back to high frequencies.

In this contribution, we analyze the modulated–demodulated control technique, emphasizing its linear time invariant nature and offering insight into the controller structure. We develop controller models, which are intended to enable the design and analysis of modulated–demodulated controllers. We briefly define negative imaginary systems and summarize the key results of negative imaginary systems theory relating to closed-loop stability, emphasizing its importance in the control of microcantilevers. We outline the implementation of positive position feedback (PPF) [9] and resonant [10] controllers, which are both effective in the Q control of a microcantilever. To conclude, we present results demonstrating the closed-loop control of a Bruker DMASP microcantilever and the resulting improvements in AFM images. Furthermore, we highlight MHz operation of the controller using a reconfigurable field programmable analog array in the baseband, confirming its ability to effectively configure the high-bandwidth controller. The modulated–demodulated control technique is well suited to the control of high-frequency resonant dynamics and we believe this technique could find application in high-speed atomic force microscopy and high-frequency MEMS.
2. Q control of an AFM microcantilever

2.1. Overview of Q control

Amplitude modulation atomic force microscopy (AM-AFM), commonly referred to as tapping mode AFM when intermittent tip-sample contact is made, has become an integral tool in the investigation of materials at the nanoscale. However, this imaging technique is not without its limitations. Cantilever dynamics play a crucial role in the system performance. Most notably, there exists an inherent tradeoff between the sensitivity and speed of the measurement [11]. The Q factor of a cantilever can be related to the damping by the expression \( Q = 1 / 2 \zeta \). High Q factors promote high sensitivity, operation in the attractive regime [12] and low tip-sample force interactions [13], suitable for the interrogation of delicate samples, such as biological materials and soft polymers, but at the expense of the imaging bandwidth. On the other hand, low Q factors enable high-speed scanning with low resolution and increased tip-sample force interactions [14]. Interestingly, Q control has been shown to promote either attractive or repulsive mode imaging, helping to eliminate the issue of bistability in AM-AFM [12,15].

Since the introduction of Q control in dynamic mode AFM [16], it has been relied upon extensively in tapping mode AFM, particularly when imaging biological samples [17–19] and imaging in liquid [20,21]. In fact, a myriad of advanced techniques have been developed, including adaptive control techniques [22–24], full state feedback control [25], optimal stochastic control [26,27], observer-based techniques [28,29], piezoelectric shunt control [30] and parametric resonance [31]. While digital controller implementations are advantageous due to their accuracy and reconfigurability, the Q control of high-frequency microcantilevers typically requires the development of high-bandwidth analog circuit implementations.

2.2. Time delay Q control and its limitations

The most commonly implemented active Q control technique is time delay Q control. A time delay, equivalent to a 90° phase shift at the resonance of the controlled mode, is applied to the measured microcantilever tip displacement signal, producing an estimate of the tip velocity [13,15]. Feedback of the estimated cantilever tip velocity enables the control of the Q factor. The widespread use of this technique is the result of the relative ease at which it can be implemented. However, owing to the multimodal nature of the microcantilever, the dynamics of the controlled mode cannot be affected without risking instability of the higher modes [32]. Furthermore, time delay Q control is unsuitable for the Q control of multiple modes, which may find important applications in the emerging field of multifrequency AFM [33].

2.3. High-speed AFM and Q control

Real-time video-rate AFM imaging is set to revolutionize the world of microscopy, enabling the observation of processes that were previously unobservable, such as the dynamics of biomolecular processes [34]. Since the maximum achievable imaging rate is proportional to the resonance frequency of the microcantilever, high-speed imaging requires the utilization of high-frequency microcantilevers [35]. Several commercially available microcantilevers, including the Nanoworld Arrow UHF and Olympus OmegaLever AC55, feature megahertz resonance frequencies and are specifically designed for high-speed imaging. However, very few Q control techniques are well suited to the control of high-frequency microcantilevers.

We intend to overcome the limitations of existing Q control techniques by focusing on the development and implementation of a reconfigurable, high-bandwidth Q control technique, specifically designed for application to ultrahigh-frequency microcantilevers, utilizing the modulated–demodulated control technique.

3. Modulated–demodulated control

Modulated–demodulated control is a periodic vibration control technique, applicable to systems possessing high-frequency resonant dynamics. The modulated–demodulated controller shown in Fig. 1 consists of two branches, each containing a demodulator with a low-pass filter \( F(s) \), an LTI controller \( C(s) \) and a modulator [6]. As is characteristic in linear modulation schemes, synchronous demodulation of the measured output signal is performed using orthogonal carrier signals

\[
s(t) = s_0 \cos(\omega_0 t) - s_0 \sin(\omega_0 t).
\]

Provided that the carrier frequency is significantly higher than the highest frequency component in the modulating signal, accurate estimates of the in-phase \( (s_0) \) and quadrature \( (s_0) \) components of the modulating signal are obtained at the filter outputs. Demodulation significantly reduces the requirements of the LTI controller and low-bandwidth compensators can be employed to process the baseband signal. The purpose of the filter \( F(s) \) is to significantly attenuate the components at \( 2\omega_0 \), while also decoupling the controller from adjacent plant modes. Modulation of the controller outputs shifts the signals back to the resonance frequency. The following requirements must be satisfied to ensure practical application of the modulated–demodulated control technique [6]:

- The half-power bandwidth of the mode is significantly lower than the resonance frequency \( (Q = \omega_0/\Delta \omega \gg 1) \), and
- \( F(s) \) is a low-pass filter with bandwidth \( \omega_0 \approx \omega_0 \).

Modulated–demodulated control is advantageous, because it significantly reduces the bandwidth requirements of the baseband controller, potentially enabling the use of low-bandwidth, reconfigurable systems, such as low-bandwidth digital controllers and the field programmable analog array (FPAA). The direct application of digital controllers to high-bandwidth systems is often limited by the accumulation of time delays. However, time delays in the baseband do not affect the bandwidth of the modulated–demodulated controller. For a detailed systems analysis of modulated–demodulated control systems, see [36,37]. A conceptual overview of the modulated–demodulated control technique is provided in Fig. 2.

4. LTI controller models

4.1. SISO transfer function

The equivalent modulated–demodulated controller in Fig. 1 can be modeled as a linear time invariant system, significantly simplifying the subsequent analysis. The ensuing derivation is intended to expose the operation of this controller architecture and offer insights into its limitations.

Amplitude modulation is a nonlinear process that can be analyzed with the Laplace transform by utilizing results from the residue theorem. The modulating property of the Laplace transform

\[
\mathcal{L}\{u(t) \cos(\omega t)\} = \frac{1}{4\pi^2} \int_C \left( \frac{1}{z-j\omega} + \frac{1}{z+j\omega} \right) U(z)dz
\]

where the Bromwich path \( C \) lies to the right of the imaginary axis.
must be adequately filtered before is the carrier frequency and orthogonal sinusoids \( \cos(\omega_1 t + \phi) \) and \( \sin(\omega_1 t + \phi) \), whose Laplace transforms are determined accordingly as

\[
\cos(\omega_1 t + \phi) = \cos(\omega_1 t) \cos \phi - \sin(\omega_1 t) \sin \phi
\]

and

\[
\sin(\omega_1 t + \phi) = \sin(\omega_1 t) \cos \phi + \cos(\omega_1 t) \sin \phi
\]

Determination of the input–output relationship of the in-phase component requires the demodulation of the input signal with \( 2 \cos(\omega_1 t + \phi) \), subsequent multiplication with the transfer functions of the demodulation filter \( F(s) \) and the baseband controller \( C(s) \) in the frequency domain, and modulation of this baseband signal with \( \cos(\omega_1 t) \), resulting in

\[
U_l(s) = \frac{1}{2} \left[ e^{j\omega_1} Y(s - 2j\omega_1) + e^{-j\omega_1} Y(s) \right] (F \cdot C)(s - j\omega_1) + \frac{1}{2} \left[ e^{j\omega_1} Y(s) + e^{-j\omega_1} Y(s + 2j\omega_1) \right] (F \cdot C)(s + j\omega_1) + \frac{1}{2} \left[ e^{j\omega_1} Y(s - 2j\omega_1) - e^{-j\omega_1} Y(s) \right] (F \cdot C)(s - j\omega_1) + \frac{1}{2} \left[ e^{j\omega_1} Y(s) - e^{-j\omega_1} Y(s + 2j\omega_1) \right] (F \cdot C)(s + j\omega_1).
\]

Evaluation of the input–output relationship of the quadrature component is performed in a similar manner, but requires the demodulation of the input signal with \(-2\sin(\omega_1 t + \phi)\) and the successive modulation of the baseband signal with \(-\sin(\omega_1 t)\), resulting in

\[
U_q(s) = \frac{1}{2} \left[ e^{j\omega_1} Y(s - 2j\omega_1) - e^{-j\omega_1} Y(s) \right] (F \cdot C)(s - j\omega_1) + \frac{1}{2} \left[ e^{j\omega_1} Y(s) - e^{-j\omega_1} Y(s + 2j\omega_1) \right] (F \cdot C)(s + j\omega_1).
\]

Summing the in-phase and quadrature components eliminates the nonlinear terms and the equivalent modulated–demodulated controller can be modeled as

\[
C_{eq}(s) = e^{-j\omega_1} (F \cdot C)(s - j\omega_1) + e^{j\omega_1} (F \cdot C)(s + j\omega_1),
\]

where \( F(s) \) is the demodulation filter, \( C(s) \) is the controller in the baseband, \( \omega_c \) is the carrier frequency and \( \phi \) is the phase between the carrier signals applied to the modulator and demodulator.

While mathematically the nonlinear terms in the preceding derivation do cancel, in practice, this cancellation is incomplete. Asymmetries between the in-phase and quadrature branches, owing to the tolerances of the electronic components, result in a nonlinear system. Practical implementations demand symmetry between the in-phase and quadrature branches. Furthermore, the frequency components at \( 2\omega_c \) must be adequately filtered before modulation, as shown in Fig. 2, to minimize the presence of harmonics and to ensure the system retains the property of linear time invariance.

### 4.2. SISO state space representation

Let the baseband system, previously referred to as \( (F \cdot C)(s) \), be described as \( G(s) \). Consider the following minimal state space representation \( \{A, B, C, D\} \) of \( G(s) \)

\[
\dot{x} = Ax + Bu
\]

\[
y = Cx + Du,
\]

where \( A \in \mathbb{R}^{n \times n} \), \( B \in \mathbb{R}^{n \times 1} \), \( C \in \mathbb{R}^{1 \times n} \) and \( D \in \mathbb{R} \).

Expressing the SISO transfer function of the modulated–demodulated controller in terms of state space matrices yields

\[
C_{eq}(s) = e^{-j\omega_1} G(s - j\omega_1) + e^{j\omega_1} G(s + j\omega_1)
\]

\[
= e^{-j\omega_1} C(s - (A + j\omega_1 I))^{-1} B + De^{-j\omega_1}
\]

\[
+ e^{j\omega_1} C(s - (A - j\omega_1 I))^{-1} B + De^{j\omega_1},
\]

where

\[
X(s) = \frac{1}{2} \left[ e^{j\omega_1} \left( \frac{1}{s - j\omega_c} + \frac{1}{s + j\omega_c} \right) + e^{-j\omega_1} \left( \frac{1}{s - j\omega_c} - \frac{1}{s + j\omega_c} \right) \right].
\]
where \( I \) is the \( n \times n \) identity matrix. This expression reveals that the modulated–demodulated controller can be represented as the parallel interconnection of two subsystems with complex-valued system matrices,

\[
\hat{x} = \begin{bmatrix} A + j\omega_{e} I & 0 \\ 0 & A - j\omega_{e} I \end{bmatrix} x + \begin{bmatrix} B \\ 0 \end{bmatrix} u
\]

\[ y = \begin{bmatrix} Ce^{-j\phi} & Ce^{j\phi} \end{bmatrix} x + 2D \cos \phi u. \tag{5} \]

The eigenvalues of the \( A \) matrix are shifted by \( \pm j\omega_{e} \), a result of the modulation, and the output \( y \) is clearly dependent upon the phase \( \phi \).

While this system representation describes the input–output dynamics of the modulated–demodulated controller, a more useful state space representation consists of real-valued system matrices. Such a representation can be obtained via the linear transformation \( T \), which is defined as

\[
T = \begin{bmatrix} I & I \\ -jI & jI \end{bmatrix}, \tag{6}
\]

where \( I \) is the \( n \times n \) identity matrix. The resulting state space representation of the controller \( \{A, B, C, D\} \) is

\[
\hat{x} = \begin{bmatrix} A & -\omega_{e} I & 0 \\ \omega_{e} I & A & 0 \\ 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 2B \end{bmatrix} u
\]

\[ y = \begin{bmatrix} C \cos \phi & C \sin \phi \end{bmatrix} x + 2D \cos \phi u. \tag{7} \]

Thus, a clear relationship between the baseband system and the resulting modulated–demodulated controller has been established. Multiple modulated–demodulated controllers could also be integrated into one single system via parallel interconnection; each equivalent controller is defined by its baseband system and corresponding carrier frequency \( \omega_{e} \) and phase \( \phi \).

4.3. Poles and zeros

Fig. 3 illustrates the migration of poles from the baseband. While the poles are shifted by \( \pm j\omega_{e} \), enabling the intuitive design of the poles of the baseband controller, the modulated–demodulated control technique places severe restrictions on the locations of the zeros in the complex plane, complicating the analysis and design of generalized controllers. Clearly, the behavior of the zeros is periodic with \( \phi \). Furthermore, the angle \( \phi \) dictates the controller’s phase at resonance. In Q control, the most useful result occurs when \( \phi = \pm \pi \) as this enables estimation of the cantilever tip velocity. Thus, since \( D = 2D \cos \phi \), \( \lim_{s \to -s} G(s) = 0 \); the resulting controller is strictly proper.

5. Negative imaginary systems

The design of robustly stable feedback controllers for highly resonant, multimodal systems is by no means straightforward. Negative imaginary systems theory has significant application in the design of robustly stable feedback controllers for highly resonant flexible structures [39]. Stability conditions for the feedback interconnection of negative imaginary systems can be concisely summarized and stability can be guaranteed, even in the presence of plant parameter uncertainties and unmodeled dynamics, as long as a DC gain condition is satisfied [40,41].

Assuming there are no poles on the imaginary axis, negative imaginary systems are stable systems, which satisfy the following property

\[
j[P(j\omega) - P^{*}(j\omega)] \geq 0 \quad \forall \omega \geq 0, \tag{8} \]

where \( P^{*} \) is the complex conjugate transpose of \( P \). Strictly negative imaginary systems must satisfy the strict inequalities. SISO systems with the property of interlacing poles and zeros and whose phase remains between 0° and 180° are negative imaginary. A more complete definition of negative imaginary systems can be found in [42].

Negative imaginary systems can also be defined in the state space domain. The minimal state space system

\[
\dot{x} = Ax + Bu
\]

\[ y = Cx + Du, \tag{9} \]

where \( A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times m}, C \in \mathbb{R}^{n\times n} \) and \( D \in \mathbb{R}^{n\times m} \) is negative imaginary if and only if [41]:

- \( A \) is Hurwitz,
- \( D \) is symmetric, and
- a positive definite matrix \( Y \in \mathbb{R}^{n\times n} \) exists, satisfying

\[
AY + YA^{T} \preceq 0, \tag{10} \]

\[ B + AYC^{T} = 0. \tag{11} \]

These conditions may conveniently be expressed with the following linear matrix inequality [41]
A more recent result utilizing spectral conditions has been developed to ascertain whether a system is negative imaginary [43]. Instead of solving a linear matrix inequality, the eigenvalues of a positive definite matrix may often be modeled to ascertain whether a system is negative imaginary [44].

Resonant systems with collocated sensor/actuator pairs can be modeled as strictly negative imaginary systems. Such systems, possessing multiple modes, can be modeled in the following manner:

\[
G(s) = \sum_{i=1}^{N} \frac{\gamma_i s^2 + 2\zeta_i \omega_i s + \omega_i^2}{s^2 + 2\zeta_i \omega_i s + \omega_i^2}; \quad \gamma_i, \zeta_i, \omega_i > 0.
\]

(13)

Resonant systems without collocated sensor/actuator pairs, such as an atomic force microscope microcantilever, may often be modeled as finite frequency negative imaginary systems and may still be analyzed using the results of negative imaginary theory [44]. Furthermore, SISO PPF and resonant controllers of the form

\[
C_{\text{ppf}}(s) = \frac{K_{\text{ppf}} s^2 + 2\zeta s + \omega_0^2}{s^2 + 2\zeta s + \omega_0^2}; \quad \gamma, \zeta, \omega_0 > 0,
\]

(14)

\[
C_{\text{r}}(s) = \frac{-\beta(s^2 + 2\zeta s + \omega_0^2)}{s^2 + 2\zeta s + \omega_0^2}; \quad \beta, \zeta, \omega_0 > 0,
\]

(15)

are strictly negative imaginary systems [41].

The positive feedback interconnection of a negative imaginary system \(M(s)\) and a strictly negative imaginary system \(N(s)\) is internally stable if and only if \(M(0)N(0) < 1, M(\infty)N(\infty) = 0\) and \(N(\infty) \geq 0\) [40]. Stability is ensured by phase stabilization, where restrictions on the phase of the two open loop systems ensure that instability is impossible. Closed-loop stability is guaranteed when strictly negative imaginary controllers are connected in positive feedback with the plant \(G(s)\) (Eq. 13) under the condition that \(C(0)G(0) < 1\). Fig. 4 illustrates this positive feedback interconnection.

Through careful selection of the baseband system, it may be possible to implement negative imaginary controllers using the modulated–demodulated control technique. Although the synthesis of negative imaginary controllers is a non-convex problem, the state space models for the modulated–demodulated controller (Eq. 7) enable the rapid testing of the negative imaginary conditions, which is useful in the design of controllers for negative imaginary systems.

6. Controller realizations

6.1. Positive position feedback control

Originally intended for application to large space structures, PPF control is well suited to the active vibration control of highly resonant, flexible structures [9]. The PPF controller is essentially a second-order low-pass filter, which ensures rapid roll-off of the controller response, minimizing spillover into higher modes. PPF control is advantageous owing to its high-frequency roll-off; closed-loop stability irrespective of actuator dynamics, which is an improvement over velocity feedback; and the fact that closed-loop stability only depends on the stiffness properties of the structure [45].

PPF control is well-documented in the existing literature and has found numerous applications in current technologies. Examples include the vibration control of a hard drive micro-actuator [46] and the accuracy and speed improvement of a commercial AFM [47]. Recent work has also demonstrated the importance of PPF control in the multimodal Q control of an AFM microcantilever [48], a result that may contribute to the development of multifrequency AFM techniques.

We have previously demonstrated the implementation of a PPF controller when the baseband system is configured with a first-order low-pass filter and a gain as illustrated in Fig. 5 [8],

\[
(F \cdot C)(s) = \frac{K_{\text{ppf}}}{s + \omega_0}.
\]

(16)

Using Eq. 2, the transfer function of the equivalent controller can be derived as

\[
C_{\text{eq}}(s) = \frac{2K_{\text{ppf}} (s + \omega_0) \cos \phi + \omega_k \sin \phi}{(s + \omega_0)^2 + \omega_0^2},
\]

(17)

where \(K\) is the DC gain of the controller, \(\omega_0\) is the bandwidth of the low-pass filter, \(\omega_k\) is the carrier frequency and \(\phi\) is the phase between the carrier signals applied to the modulators and demodulators. Selection of \(\phi\) gives the user specific control over the controller’s phase at resonance. By setting \(\phi = \frac{\pi}{2}\), the transfer function simplifies to that of a PPF controller,

\[
C_{\text{ppf}}(s) = \frac{2K_{\text{ppf}} \omega_k}{(s + \omega_0)^2 + \omega_0^2}.
\]

(18)

To ensure that the PPF controller implementation remains robust to small changes in \(\phi\), consider the first order Taylor series expansion of Eq. 17 evaluated at \(\phi = \frac{\pi}{2}\).

\[
C_{\text{eq}}(s) \approx C_{\text{eq}}(s)|_{\phi=\frac{\pi}{2}} + \left. \frac{\partial C_{\text{eq}}(s)}{\partial \phi} \right|_{\phi=\frac{\pi}{2}} \Delta \phi
\]

\[
\approx 2K_{\text{ppf}}[(\Delta \phi) + (\omega_0 - \omega_0 \Delta \phi)]
\]

(19)

Since \(\omega_0 \ll \omega_k\) and \(\Delta \phi \approx 0\), it is clear that the PPF controller is insensitive to small changes in \(\phi\). A high-frequency zero,

\[
z = \frac{\omega_k - \omega_0 \Delta \phi}{\Delta \phi}
\]

(20)

does appear at \(\phi \neq \pm \frac{\pi}{2}\), but when \(\Delta \phi \approx 0\) this is well beyond the bandwidth of the system.

In [6], the authors implemented a multimodal modulated–demodulated controller and demonstrated its ability to suppress the first two resonant modes of a cantilever using collocated piezoelectric sensors and actuators. Fourth order Butterworth filters were used to decouple adjacent resonant modes. One drawback to the use of high-order filters in the baseband is that they introduce additional dynamics into the closed-loop system. As the gain was increased, the resonant modes of the plant were damped, but additional resonant peaks appeared as a result of the additional controller poles moving towards the imaginary axis. With the modulated–demodulated PPF controller there are only two resonant poles and no such behavior occurs.

Given a negative imaginary plant with DC gain \(\gamma\), the results of negative imaginary systems theory ensure stability of the closed
loop provided that the DC gain condition is satisfied. This translates to the following bounds on the controller gain

\[ 0 < K < \frac{\omega_b^2 + \omega_c^2}{2\omega_b\omega_c} \approx \frac{\omega_b}{2\omega_b}. \]  

(21)

Given that there is no stability guarantee when utilizing the time delay Q control technique, modulated–demodulated PPF control offers a clear advantage over time delay Q control.

6.2. Controller parameterization

While it is possible to realize alternative, and arguably simpler, high-bandwidth PPF controllers, such as the Sallen Key active filter, the parameterization of the modulated–demodulated PPF controller significantly simplifies the tuning process. Comparison of the PPF controller in Eq. 14 with the modulated–demodulated PPF controller in Eq. 18 gives

\[ C(s) = \frac{K\omega_b^2}{s^2 + 2\omega_c s + \omega_c^2} = \frac{2K\omega_b\omega_c}{s^2 + 2\omega_b s + (\omega_b^2 + \omega_c^2)}. \]  

(22)

Since \( \omega_c \gg \omega_b \), \( \omega_b \approx \omega_k \) and the resonance frequency of the PPF compensator is essentially determined by the carrier frequency. In an equally simple manner, \( \omega_b = \omega_k \). Increasing the bandwidth of the low-pass filter in the baseband results in a more damped characteristic; the controller operates over a wider range of frequencies around the resonance at the expense of increasing the bandwidth requirements of the baseband system. In contrast, a Sallen Key implementation of the PPF controller would require the simultaneous tuning of multiple circuit components, which are nonlinearly dependent on the controller parameters. The ability to independently tune the resonance and damping of the modulated–demodulated controller, through selection of \( \omega_c \) and \( \omega_b \) respectively, is thus advantageous.

6.3. Resonant control

Designed specifically to take advantage of sensor/actuator collocation, the application of resonant controllers results in attractive stability properties of the closed loop system [10]. Initially, resonant controllers were implemented as high Q RLC circuits with a high-pass characteristic, whose resonances were aligned to the modal resonances of the mechanical system. The 90° phase shift at resonance equates to the behavior of a differentiator, closely resembling velocity feedback control, but with superior robustness to unmodelled dynamics.

Resonant control has also found numerous applications in emerging technologies. While PPF control offers a higher level of damping performance [49], resonant control is particularly useful in applications which take advantage of its high-pass characteristic. Resonant controllers have been successfully implemented to overcome the 1/f noise of MEMS-based nanopositioners, also simplifying the controller design process [50]. Resonant controllers have also been utilized in a variety of structural damping control applications [51,52]. Furthermore, resonant control has been shown to complement PPF control in the multimodal Q control of a microcantilever [48].

The direct implementation of a resonant controller is not possible using the modulated–demodulated controller architecture. For resonant control, we require that \( \phi = \frac{\pi}{2} \) to ensure the –90° phase shift at resonance, and \( D=0 \). Since \( D = 2D\cos \phi \), these conditions cannot be achieved simultaneously. However, it is possible to implement a feedthrough term external to a modulated–demodulated PPF controller, as illustrated in Fig. 6. While initially it was proposed to introduce the feedthrough into the baseband to simplify the implementation, this increased the bandwidth requirements in the baseband; experiments indicated that while the correct behavior was achieved at resonance, it was degraded at frequencies further from the resonance.

The modulated–demodulated resonant controller, consisting of a PPF controller and a feed through, can be expressed as

\[ C_{\text{mp}}(s) = -K_1 + \frac{2K_2\omega_b\omega_c}{(s + \omega_b)^2 + \omega_c^2}. \]  

(23)

This controller becomes

\[ C_{\text{mp}}(s) = -\frac{K_1(s + 2\omega_b)}{(s + \omega_b)^2 + \omega_c^2} \]  

(24)

when

\[ K_2 = \frac{\omega_b^2 + \omega_c^2}{2\omega_b\omega_c} = \frac{1}{2} \left( \frac{\omega_b}{\omega_c} + \frac{\omega_c}{\omega_b} \right). \]  

(25)

Given a negative imaginary plant \( G(s) \), the stability of the closed loop can be guaranteed when the DC gain condition \( G(0)C(0) < 1 \) is satisfied. Since the resonant controller has a zero at DC, stability is ensured when \( K_1 > 0 \).

Several comments must be made regarding the implementation of the resonant controller. Firstly, \( \omega_b \) should not be too small, otherwise large gains are required for the PPF controller. When \( \omega_b \approx \omega_k/10 \), requiring that \( K_2 \approx 5K_1 \), good damping performance is achieved. Secondly, implementation of the modulated–demodulated resonant controller requires the ratio of two gains to be carefully set. Consider a small perturbation \( \delta \), such that

\[ K_2' = K_2 \left( 1 + \delta \right), \]  

(26)

then

\[ C_{\text{mp}}(s) = -\frac{K_1(s + 2\omega_b)}{(s + \omega_b)^2 + \omega_c^2} + \frac{\delta \omega_b^2}{(s + \omega_b)^2 + \omega_c^2}. \]  

(27)
Clearly, $K_1$ and $K_2$ must be set accurately to ensure behavior as a resonant controller and to minimize the PPF controller that results due to the mismatch in gains. While sensitive to small variations, it is straightforward to tune the resonant controller by tuning $K_1$ or $K_2$ such that the controller output to a DC input is minimized, or the controller phase is $-90\degree$ at low frequencies.

6.4. Controller design

Since there are not enough controller parameters to place all four closed-loop poles, nonlinear optimization routines were utilized to position the real components of the closed-loop poles or to minimize the $H_\infty$ norm of a particular mode using the parameters $K$, $\omega_c$, and $\omega_b$. Algorithms were developed for both PPF and resonant controllers, utilizing the $fmincon$ function in MATLAB with the following cost functions. To set a specific Q factor, the following cost function can be utilized

$$J_b = \sqrt{\sum_{i=1}^{N} (\sigma_{i,\text{des}} - \mathcal{R}(p_{i,\text{cl}}))^2},$$

where $\sigma_{i,\text{des}}$ is the real component of the desired location of the $i$th pole and $\mathcal{R}(p_{i,\text{cl}})$ is the real component of the actual location of the $i$th pole in the current iteration. To significantly attenuate a resonant mode, the following cost function can be employed

$$J_b = \| \mathbb{G}(j\omega) \|_{H_\infty},$$

where $J_b$ is the $H_\infty$ norm of the closed-loop system, which consists of the single resonant mode to be suppressed.

With a good initial guess, where the carrier frequency is close to the resonance of the mode $\omega_c \approx \omega_b$ and the baseband bandwidth is small compared to the carrier frequency $\omega_b \approx \omega_c/10$, the optimization routines converged quickly, returning the parameters $K$, $\omega_c$ and $\omega_b$ to tune the controller.

6.5. High-order controllers

While PPF and resonant control are exceptionally useful low-order control techniques, the analytical models developed in the preceding section can be used to determine the controller model for any baseband system. High-order baseband systems ensure aggressive roll-off away from the resonance frequency and can be designed to target specific modes with minimal impact on adjacent modes. These models are required to account for the closed-loop instabilities observed in [6].

7. Controller implementation

7.1. Modulated–demodulated controller architecture

Previously, we utilized the ON Semiconductor MC1496 balanced modulator/demodulator in our implementation of the modulated–demodulated controller [8]. While we successfully demonstrated operation of the modulated–demodulated PPF controller, we noted a number of limitations, namely

- the double balanced Gilbert cell architecture requires external biasing, limiting controller tunability,
- low carrier levels of 60 mVpp required for optimal carrier suppression increase susceptibility to noise. Higher carrier levels resulted in noticeable carrier feedthrough, and
- complex circuit configurations owing to required signal conditioning between stages.

To overcome the limitations of the MC1496 and simplify the controller implementation, the Analog Devices AD835 analog multiplier was utilized. The AD835 is suitable for high-bandwidth multiplication. The four-quadrant, voltage output amplifier linearly multiplies its two inputs and has a $-3$ dB bandwidth of 250 MHz. Notable improvements of the AD835 over the MC1496 include reduced harmonic distortion, reduced input feedthrough and no required external circuitry. Rated harmonic distortion levels are $-70$ dB and $-65$ dB for the second and third harmonics respectively and the $-60$ dB input feedthrough from the full-range $Y$ input is a significant improvement over the MC1496. A host of high-performance operational amplifiers were utilized to complement the AD835 multiplier and maximize the bandwidth of the controller, ensuring MHz operation was possible. A custom printed circuit board was designed for the controller as shown in Fig. 7.

7.2. Baseband controller

The Anadigm AN221E04 field programmable analog array (FPAA) was employed in the baseband to realize the PPF and resonant controllers. The FPAA utilizes switched capacitor filters to implement a range of complex analog functions. Switched capacitor filters offer comparable performance to analog filters, but can be easily implemented on an integrated circuit. Since there is no signal quantization, these filters offer excellent dynamic range.

One significant advantage of switched capacitor filters over analog filters is that precision capacitors are not required to set the filter cutoff frequency. Instead, the ratio of the capacitances and the switching frequency are the determining factors. Therefore, utilization of the FPAA ensures symmetry between the in-phase and
quadrature branches of the modulated–demodulated controller. The FPAA can be used to implement filters up to 400 kHz; beyond this range it can be employed in the baseband to extend its control capabilities.

8. Experimental results

8.1. Measurement procedure

The frequency response functions have been measured using the Zurich Instruments HF2LI Lock-In Amplifier. Since lock-in techniques determine the magnitude and phase of the system output at the frequency of the input excitation, this measurement procedure ignores nonlinear circuit operation. However, given adequate attenuation of the frequency components at $2\omega_c$ before modulation, higher harmonics are minimal and the predominant controller behavior is linear as confirmed by inspection of the FFT spectrum during measurements.

8.2. Cantilever characterization

A Bruker DMASP microcantilever (see Fig. 8) was used to demonstrate the performance of the modulated–demodulated controller. The DMASP microcantilever has a nominal fundamental resonance at 50 kHz and a second resonance at 210 kHz. The fundamental mode can be modeled as

$$G(s) = \frac{1.063 \times 10^9}{s^2 + 1346s + 1.379 \times 10^{11}},$$

where $f_0 = 59.1$ kHz, $\zeta = 0.0018$ and $Q = 276$.

The DMASP microcantilever utilizes integrated piezoelectric actuation to excite the resonant modes directly, thus eliminating dynamics from conventional piezoelectric stack actuators and the cantilever holder [53], which is essential for closed-loop control. Currently there are no commercially available ultrahigh-frequency cantilevers with integrated actuation.

8.3. Closed loop control

The results for Q factor reduction are shown in Figs. 9 and 10. The Q factor is reduced from 276 to 26 and 23 with the PPF and resonant controllers respectively. High controller gains enable significant damping of the resonance. The potential of this control technique can be demonstrated by configuring modulated–
demodulated PPF and resonant controllers with resonances of 1.2 MHz. The FPAA cannot be used to directly implement filters with cutoff frequencies greater than 400 kHz. However, as the baseband controller, the FPAA offers the advantages of a low-bandwidth, reprogrammable controller, and the modulated–demodulated control architecture extends its capabilities to the high-frequency regime. In Fig. 11, the Q factor is increased from 276 to 2180 using the PPF controller.

8.4. Baseband dynamics

The dynamics in the baseband have a profound effect on the controller’s behavior. The maximum switching frequency of this FPAA is 4 MHz and the high-frequency dynamics of the FPAA do affect the controller’s response. In Figs. 9 and 10, there is a small, but observable blip at 2.8 MHz, and also at 5.2 MHz. This is due to the 4 MHz switching frequency of the switched capacitor filters and the modulation (4 ± 1.2 MHz). While digital controllers were not tested specifically, high-order antialiasing filters will affect the controller, increasing its complexity. It is therefore recommended to keep the baseband bandwidth low and utilize a first-order low-pass filter as the antialiasing filter; the rest can be implemented digitally. DC offsets in the baseband enable the direct feedthrough of the carrier signal, thus introducing dynamics near the resonance. DC offsets were minimized using DC supplies to counteract the offsets introduced by the FPAA in the in-phase and quadrature branches. While time delays in the baseband do not affect the bandwidth of the controller, they are undesirable and in extreme cases may cause malfunction of the controller.

Fig. 10. Measured (−−) and simulated (−) frequency responses of the modulated–demodulated resonant controller (f_c = 59 kHz and f_b = 5 kHz) (left), (f_c = 1.2 MHz and f_b = 100 kHz) (right) and the closed loop frequency response of a Bruker DMASP microcantilever with the resonant controller (center). The Q factor was reduced from 276 to 23 (−–).

Fig. 11. Open (−) and closed loop (−−) frequency responses of a Bruker DMASP microcantilever with the modulated–demodulated PPF controller (f_c = 59 kHz and f_b = 5 kHz). The Q factor has been enhanced from 276 to 2180.

Fig. 12. Comparison of images with no Q control (39.1 μm/s) (left) and with Q control (78.2 μm/s) (right) of the TGZ3 calibration grating. The Q factor was reduced from 221 to 56.
9. AFM scans

To demonstrate operation of the modulated–demodulated controller in amplitude modulation AFM, images were obtained using an NT-MDT Ntegra atomic force microscope and the Bruker DMAP microcantilever. An NT-MDT TG23 calibration grating, featuring step heights of 560 ± 4 nm with a periodicity of 3 ± 0.05 μm, was used. The scan size was set to 10 μm × 10 μm and the amplitude set point was 50%. The results are highlighted in Fig. 12.

Without Q control, the microcantilever was scanned across the sample’s surface at 39.1 μm/s. Upon encountering a large down-wards step, the probe loses contact with the sample. Using the modulated–demodulated controller, the Q factor was reduced from 221 to 56. The scan speed was doubled to 78.2 μm/s, accompanied by a noticeable improvement in the estimated sample topography at the expense of increased tip-sample force interactions [14].

Owing to the dynamics of the multipliers and DC offsets in the baseband, which result in carrier feedthrough into the controller output signal, high controller gains resulted in image artefacts. Such issues pose practical limits on the obtainable performance of the modulated–demodulated controller.

10. Conclusions

We have outlined the application of modulated–demodulated control to the Q control of a microcantilever in amplitude modulation atomic force microscopy. Using the modulated–demodulated control technique, it is possible to implement high-bandwidth PPF and resonant controllers using low-bandwidth reconfigurable controllers in the baseband. Both PPF and resonant controllers are particularly effective in the control of negative imaginary systems. Furthermore, using the state space model of the modulated–demodulated controller, it is straightforward to design and analyze more complex baseband controllers. The controller developed is capable of operating in the MHz range and was designed to address limitations in the Q control of high-frequency AFM microcantilevers. We believe that the modulated–demodulated control technique could find application in high-speed atomic force microscopy and high-frequency MEMS.

References


